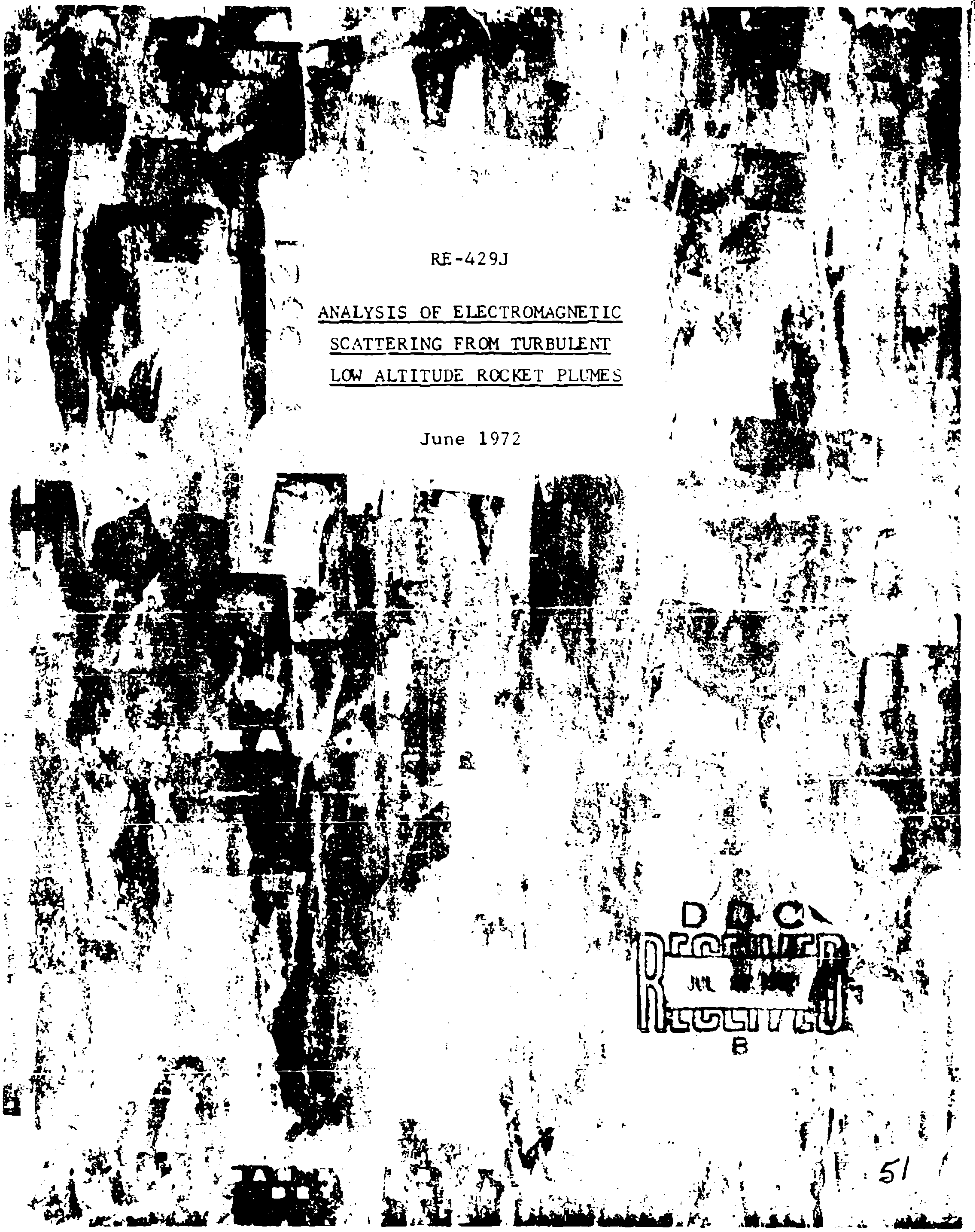

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ANALYSIS OF ELECTROMAGNETIC SCATTERING FROM TURBULENT LOW ALTITUDE ROCKET PLUMES

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A theoretical study has been made of the scattering of electromagnetic waves from both underdense and overdense turbulent low altitude rocket plumes. The analysis considers a number of parametric assumptions about the nature of the turbulence, namely, type of correlation function, correlation lengths, isotropy of turbulence, finite conductivity and plume intermittency. The analytical results have been applied to the cross section of both an underdense and overdense plume at 65,000 ft altitude. The most important result is that the theoretical cross section can be large for some reasonable suumptions about the nature of the plume turbulence.

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ABSTRACT

A theoretical study has been made of the scattering of electromagnetic waves from both underdense and overdense turbulent low altitude rocket plumes. The analysis considers a number of parametric assumptions about the nature of the turbulence, namely, type of correlation function, correlation lengths, isotropy of turbulence, finite conductivity and plume intermittency. The analytical results have been applied to the cross sections of both an underdense and overdense plume at 65,000 ft altitude. The most important result is that the theoretical cross section can be large for some reasonable assumptions about the nature of the plume turbulence.

INTRODUCTION

Analytical and experimental study of the effects of radar reflection from plumes has been an area of active research for many years. In this paper we concentrate on the analytical aspects of the problem. Our plan is to formulate models that bear a reasonable resemblance to a practical plume scattering problem and yet are simple enough to handle. We will introduce into the models some scattering phenomena that have not been introduced into practical rocket plume-EM wave scattering problems before.

We do not mean to say that these are the only phenomena that should be considered in the electromagnetics of plume scattering. They are not. In fact, the numerical results of this paper (i.e., the calculated cross sections) vary over a wide range of values depending upon the nature of the model of the scattering process. However, if we are to make progress in this very complicated field we must choose a large number of models for our theoretical analysis to compare with experimental data. By a long and tedius process one will arrive at models that correctly describe the electromagnetics under all operating conditions.

The analytical studies of the problem have in general been concerned with two main areas of interest, namely fluid mechanics and electromagnetics. The fluid mechanics approach considers

the problem of determining the electron density, density fluctuation, temperature and collision frequency distribution throughout the plume. Many analytical models have been concerned with the various gas dynamic effects that could influence the electron properties, some of which are both equilibrium and nonequilibrium chemistry, turbulence in the plume core, viscous interaction effects, diffusion of the plume into a moving environment, temperature differences, and inhomogeneity.

The study of the electromagnetic effect has generally been separated into two categories. The first is the volume scattering problems where the illuminating energy to first order passes unattenuated through the plume. The scattered energy results from the electrons excited by this energy. Secondly, the problems can be considered as surface scattering problems where the plume electron concentration is high enough that one can define a surface like a metal plate from which scattering will take place.

In this paper we will concentrate on the study of the electromagnetic aspects of the problem. We will briefly describe the analysis that was made on the fluid mechanics aspects and show how the results of this turbulent calculation were used as a "take off" point for the description of the analytical models of the turbulent plume used in the EM wave scattering analysis.

PLUME CALCULATION FOR VOLUME SCATTERING PROBLEM

Two computer programs were used to determine the electron distribution in the plume. The first, a nozzle program, calculated the ionized and neutral species concentration in the combustion chamber and the nozzle. The most important ionizing species of the liquid propellant are the alkali metal impurties of sodium and potassium. Their molar concentration is about 10 ppm. A second computer program is used for the fluid analysis of the rocket exhaust plume. The species concentration at the nozzle exit plane, as calculated by the previous program, is used as initial condition for this program. The program considers turbulent mixing and nonequilibrium chemistry in the calculation of the neutral and ionized species concentrations throughout the plume that is flowing into a moving environment. The plume program is a Grumman adaptation of an existing program (Ref. 1). The distribution of the electron concentration obtained from the exhaust plume computer calculation is used to construct an analytical model of the plume that has a relatively simple geometric shape.

We have sketched the actual electron concentration of a liquid propellant axisymmetric plume from the computer print-out of the exhaust plume program (Fig. 1). These results are for an altitude of 65,000 feet. Although not shown on the figure, the overdense region

of the plume for the frequencies considered is very small; hence scattering from this region will be neglected. On the other hand, there appears to be a very large region of weakly ionized gas extending as far as 150 meters from the nozzle exit, with a maximum radius of about 10 nozzle exit radii. From this observation the first conclusion that we can draw about this liquid propellant plume is that it will be underdense. Therefore we must consider a volume scattering analysis. Further investigation shows that the electron density varies along the axis from approximately 10^{12} at the nozzle exit plane, decreasing very rapidly at first, and reaching 10^9 at about 150 meters from the nozzle exit plane. The radial variation at every axial position varies very slowly from a maximum value on the axis out to some radial position and then drops sharply to a relatively low value. The curve has a knee shape. As an example of this, consider the radial variation at an axial location of 28 meters. The electron density is 1.3×10^{10} on the centerline, 10^9 at five nozzle radii and drops sharply to 1.8×10^6 at 6.25 nozzle radii. The radial position of the knee varies in an approximately conical manner from the nozzle exit plane to an axial position about 28 meters from the nozzle exit plane. The position of the knee remains relatively constant at about 10 nozzle radii from that point to

150 meters from the nozzle exit plane. One could consider the electron distribution as contained in a cylinder of radius 10 nozzle exit radii and 150 meters long, with the electron density decreasing monotonically from 10^{12} to 10^{9} along the axis. For the volume scattering problem we have taken the model of the turbulent plume to be a cylinder 150 meters by 7.5 nozzle exit radii in radius with an average uniform electron distribution. The axial variation of the electron concentration has been neglected.

ELECTROMAGNETICS OF VOLUME SCATTERING

Volume scattering of electromagnetic waves from turbulent fluid plasmas was first considered by Booker and his associates (Ref. 2) in the early Fifties and has been considered by others in a similar manner since that time (Refs. 3 and 4). Physically, Booker's assumption is that the plasma is essentially transparent to the primary radiation from the external source. The scattered or reradiated energy is generated by the excitation of the electrons in the turbulent plasma by this primary radiation. The mathematical technique describing this scattering mechanism is called the Born approximation and was first developed to describe quantum mechanical scattering phenomena. Under the assumption of the Born approximation (where multiple scattering is not important) we can write down the integral for the scattered power, which is essentially nothing more

than a solution to Maxwell's equations for the scattered radiative power for a collection of sources (excited electrons) in some volume in space. The cross section, σ, represents the ratio of the scattered power in a certain direction to the incident power. This integral is

$$\sigma = \sin^2\Omega\, \gamma_e^2 \int_{V_1}\int_{V_2} \langle N_1 \;\; N_2 \rangle \cdot e^{-(\vec{k}_i - \vec{k}_r)\cdot(\xi^{(1)} - \xi^{(2)})}\, dV^{(1)} dV^{(2)} \quad (1)$$

The time average of the electron density at two points appears within the integral

$\vec{k}_i, \vec{k}_r$ = incident, reflected wave vectors

γ_e = classical electron radius

Ω = angle between the incident electric field vector and the scattered wave vector

$V^{(1)}, V^{(2)}$ = volume; superscripts represent two independent integrations

The term $\langle N_1 \;\; N_2 \rangle$ can be expanded into the form

$$\langle N_1 N_2 \rangle = N_{10} N_{20} + \langle \Delta N_1 \; \Delta N_2 \rangle \quad (2)$$

where N_{10}, N_{20} are the time average or mean values of the electron concentration at the point 1,2, and $\Delta N_1, \Delta N_2$ are the fluctuating values of the electron concentrations. When Eq. (2) is substituted into Eq. (1), the scattering cross section is composed of two terms: the first is the scattering from the mean electron concentration, called the coherent scattered energy; the second is the scattering from the fluctuating part, called the incoherent scattered energy. We find that in most rocket plume-EM wave volume scattering problems where the fluid flow is turbulent, the incoherent scattered energy is by far the greater.

The usual procedure in calculating the volume scattering cross section of the turbulent body of ionized gas is to introduce the two point autocorrelation functions of the turbulence for the term $\langle \Delta N_1 \Delta N_2 \rangle$ which can be integrated, either numerically or analytically. The form of the autocorrelation function is such that it decreases monotonically as the distance increases between the spatial points of ΔN_1 and ΔN_2. The characteristic length of this decay is called the correlation length. The autocorrelation function has the form

$$\langle \Delta N_1(x_1) \Delta N_2(x_2) \rangle = \langle (\Delta N_1)^2 \rangle f(x_1 - x_2) \quad . \tag{3}$$

An example is the gaussian

$$f(x_1 - x_2) = e^{-\frac{(x_1 - x_2)^2}{2a^2}} \quad .$$

where a is the correlation length. In the volume scattering problem treated here we will consider only isotropic turbulence.

Therefore the function f will vary only as the magnitude of the distance between the spatial points.

The usual procedure in volume scattering problems is to assume that the characteristic length of the turbulence is much smaller than the smallest linear dimension of the plasma. If one considers this case and substitutes Eq. (3) into Eq. (1) (considering the incoherent scattering only) and integrates over the volume of the ionized gas one finds

$$\sigma = (\sin^2\Omega)(\eta_\epsilon^2)V\ddagger(f,a) \quad . \tag{4}$$

In Eq. (4) we have substituted a discrete function for the autocorrelation function f (for example, the gaussian or the exponential). Since the correlation function is expressed in terms of a difference between two points in a gas, a coordinate transformation has been made substituting sum (x_2^1) and difference x_1^1 for the real coordinates

$$\begin{aligned} x_1^1 &= x_1 - x_2 \\ x_2^1 &= \tfrac{1}{2}(x_1 + x_2) \quad . \end{aligned} \tag{5}$$

Because the correlation length is small relative to the plasma size, the function f dies out very fast for large values of the difference coordinates. Therefore the limits of integration for

the difference coordinates can be replaced by $\pm\infty$. This integral then simply becomes the three dimensional spectral density $\Phi(f,a)$ of the particular autocorrelation function f with its particular correlation length a. Equation (4), sometimes called the Booker-Gordon relation, is the conventional cross section relation to use when volume scattering in the Born approximation is applicable and when the scale of the turbulence is small compared to the size of the plasma.

However, recent experimental measurements indicate that the turbulence scale may not always be small compared to the size of the plume. Slattery (Refs. 5 and 6) and his associates at Lincoln Laboratory have made microdensitometer tracings from schlieren photographs of light passing through the hypersonic turbulent wake of spheres fired in a ballistic range. Using a mathematical technique developed by Uberoi and Kovasznay, they were able to reduce the correlation in the contrast on the photographic plate to an actual autocorrelation function of the turbulence in the wake. These results show that the correlation length (defined as the distance required for the correlation function to reduce in value 1/e) is constant and approximately one-half the diameter of the sphere generating the wake. Figure 2 is taken from these results and shows that the mean values of the wake diameter to correlation length ratio can vary from 1 at 10 sphere diameters behind the

wake to 10 at 1000 sphere diameters behind the wake for a 3/8 in. sphere.

Measurements of statistical turbulence properties have been made at Stanford Research Institute (Ref. 7) by seeding an ethylene oxygen flame with potassium chloride. In this case, local measurements were made of the fluctuation of the ion and electron density with ion and electron probes. Correlation coefficients were determined from the probe measurements by a signal correlator. The results of their data taken from the report are shown in Fig. 3. Mean values of the plume diameter to correlation length ratio are 4.0 at 9.0 nozzle diameters downstream and 5.3 at 20.0 nozzle diameters downstream of the nozzle exit plane. The above two experiments indicate that under some physical conditions the turbulence scale lengths of wake and jets may not be small compared to the size of the exhaust plume. This must be considered in our electromagnetic scattering analysis. A full discussion of the analysis for this case is presented in Ref. 8. We will sketch only the important highlights here, showing that for a large correlation length the finite volume cross sections can be large.

The finite volume effect is easily seen in a one dimensional problem. Substituting Eq. (3) in the incoherent scattering part of Eq. (1) in one dimension only (Fig. 4), we find that the collision cross section can be written in the form

$$\sigma = 2A \int_{-2A}^{2A} g(x_1^1)\, e^{-ikx_1^1} \left[1 - \frac{x_1^1}{2A}\right] dx_1^1$$

$$k = |\vec{k}_i - \vec{k}_r| \tag{6}$$

where $g(x_1^1) = \langle (\Delta N)^2 \rangle f(x_1^1)$, the turbulence correlation function and suppressing $\sin^2\Omega\, \gamma_e^2$. The x_1^1 is the difference coordinate [as in Eq. (5)]. Use of sum and difference coordinates amounts to a rotation of the coordinate system (Fig. 5). Integration will proceed from line 4 to line 1 and from line 3 to line 2 in the x_2^1 coordinate. The lines are then summed in the x_1^1 difference coordinate. If we let $x_1^1 = ay$, Eq. (6) becomes

$$\sigma = 2A \int_{-2A/a}^{2A/a} a g(y)\, e^{-ikay} \left[1 - \frac{y}{2A/a}\right] dy \quad . \tag{7}$$

If we now let $A/a \rightarrow \infty$, the second term in the bracket approaches zero (A/a is in the denominator) and the first integral takes the form

$$\sigma = 2A \int_{-\infty}^{\infty} g(x^1)\, e^{-ikx^1} dx^1 \quad . \tag{8}$$

Equation (8) is the one dimensional Booker-Gordon equation, the integral being the spectral density of the autocorrelation function and 2A being the one dimensional "volume." However, when A/a is finite we see that an extra term appears in the scattering integral [Eq. (7)]. Under these conditions, incorrect results would be obtained if the Booker-Gordon integral were .

The problem of evaluating the scattering cross section for a three dimensional finite volume plasma is a generalization of the evaluation of the scattering cross section for the one dimensional plasma. A set of transformation equations is introduced [similar to Eq. (5)] for each of the three real space coordinates. Since we have to integrate over the volume twice, as in the one dimensional problem, the integration will be over a six dimensional volume. The scattering cross section will be represented by

$$\sigma = \sin^2\Omega \; \gamma_e^2 \iint g(x_1^1, y_1^1, z_1^1) \; e^{i\vec{k}\cdot\vec{x}} \, dx_1^1 \, dy_1^1 \, dz_1^1 \, dx_2^1 \, dy_2^1 \, dz_2^1 \quad . \tag{9}$$

The subscripts 1,2 are the difference and sum coordinates, respectively, as in Eq. (5). The limits of integration will contain the surface of the plasma as in the one dimensional scattering relation. Since we are considering a homogeneous turbulence, the

correlation length will be a function of the difference coordinates only. If we consider the integration in our one dimensional plasma, again we see that it is really in two regions of the transformed space (Fig. 5). The limits of integration for the double integral are different in each region. Therefore the one dimensional scattering cross section is really a sum of two integrals in transformed space. For the three dimensional plasma the scattering cross section will be a sum of eight integrals over eight regions in the six dimensional space. We can write this sum of eight integrals as

$$\sigma = \sin^2\Omega \;\, \gamma_e^2 \sum_{i,j,k=1}^{i,j,k=2} I_{kji} \; . \tag{10}$$

One of the integrals can be written as

$$I_{121} = \iint_4^1 \iint_3^2 \iint_4^1 h \; dx_2^1 \; dx_1^1 \; dy_2^1 \; dy_1^1 \; dz_2^1 \; dz_1^1 \tag{11}$$

where the upper limits are the subscripts of I_{kji} in Eq. (10). The numbers over the integrals represents the limits of integration from, say, line 4 to line 1 in the x and z coordinates and line 3 to line 2 in the y coordinate. The integration proceeds in the order x,y,z. In the one dimensional case the equation for lines 1 through 4 contains the parameter $\pm A$, the boundary of the plasma. In the case of the three dimensional plasma, lines 1

through 4 also contain the boundary of the plasma; however, they are now in a functional form. They are called $S_{\ell mn}$ where ℓ, m, n vary from 1 to 2 and are known functions of the geometry of the plasma (Ref. 8). The equations for the limits of integration are numbered in a similar manner as for x as shown in Fig. 5. Therefore we see that there are two integration regions for each coordinate, which make a total of eight combinations of six dimensional integrations; thus the eight regions in the six dimensional space.

The actual limits of integration are given in Ref. 8, and a table of their values is reproduced in Fig. 6. We might note here that the eight integrals will contract to the single Booker-Gordon scattering integral when the correlation length is small compared to the smallest characteristic dimension of the plasma. The above relation holds for any shaped three dimensional plasma.

The Booker-Gordon analysis shows that the cross section is independent of the shape and the orientation of the plasma relative to the transmitter and receiver, but depends only on the plasma volume. The finite volume analysis shows that the orientation and the shape of the plasma influence the value of the scattering cross section.

In theory it is possible to calculate the scattering cross section of any homogeneous plasma shape, with any correlation

function by means of Eq. (10) with the limits of integration given in Fig. 6. However, the evaluation of the eight integrals for a correlation function that is not separable could be a lengthy process. The first finite volume that was considered was a parallelepiped plasma (Fig. 7) using a gaussian correlation function. Under these conditions the $S_{\ell mn}$ are constants (the boundaries of the plasma, i.e., $-A$, A, $-B$, B, $-C$, C). The gaussian correlation function is separable in the $x,y,$ and z coordinates. Since the limits of integration for the boundaries are constants and the correlation function itself is separable, it is possible to perform the integration first in the x sum and difference coordinate, then in the y sum and difference coordinate, and finally in the z sum and difference coordinate. Then each of the sum coordinates can be integrated so that the cross section is expressed as a product of three single integrals in the $x,y,$ and z difference coordinate. We can write the cross section as

$$\sigma = K_1 K_2 K_3 \tag{12}$$

where

$$K_1 = \int_{-2A}^{2A} \left(1 - \frac{x_1^1}{2A}\right) g(x_1^1)\, e^{-ikx_1^1}\, dx_1^1 \quad .$$

There are similar expressions for K_2 and K_3 with the limits of integration being $-2B$, $2B$ and $-2C$, $2C$, respectively. We can

see that the cross section for the rectangular parallelepiped is just a product of three one dimensional cross sections. We can understand the physics of the scattering mechanism by studying just one one dimensional cross section. If we consider the gaussian correlation function

$$f(x_1^1) = e^{-\frac{(x_1^1)^2}{2a^2}}$$

and make the substitution

$$y = \frac{x_1^1}{\sqrt{2}a} ,$$

the relation for K_1 becomes

$$\sigma_{FV} = 2^{3/2} Aa \left[2 \int_0^{\beta} (1 - \frac{y}{\beta}) \, e^{-y^2} \cos \sqrt{2}\, kay \, dy \right] \qquad (13)$$

where $\beta = \sqrt{2}A/a$ and the one dimensional Booker-Gordon relation is

$$\sigma_{BG} = 2^{3/2} Aa\pi^{\frac{1}{2}} \, e^{-\frac{a^2k^2}{2}} . \qquad (14)$$

The ratio of finite volume to Booker-Gordon cross section is

$$R = \frac{\sigma_{FV}}{\sigma_{BG}} = \left(\frac{1}{\pi}\right)^{\frac{1}{2}} e^{\frac{(ka)^2}{2}} \int_0^{\beta} 2(1 - \frac{y}{\beta}) \, e^{-y^2} \cos \sqrt{2}\, kay \, dy . \qquad (15)$$

Equation (15) has been solved numerically on a computer and the results have been plotted (Figs. 8 and 9). It can be seen from the figures that for $ka > 3$, the cross section can be very large compared to the Booker-Gordon values for small values of A/a.

VOLUME SCATTERING FROM A PLUME AT 65,000 FEET

The fluid mechanic and electromagnetic techniques outlined in the previous section have been applied to the problem of determining the backscatter scattering cross section of a turbulent rocket exhaust plume at 65,000 ft altitude. The turbulent plume model considered was 150 meters long with a radius of 7.5 nozzle radii. The turbulence was considered to be isotropic and homogeneous with a gaussian correlation function. For the finite volume correction to the scattering cross section we use a rectangular parallelepiped with a side dimension of 11 meters ($2A = 11$ m). Calculations were made with three assumed values of the plume correlation lengths of 0.0055, 0.055, and 0.55 m. The corresponding values of A/a are then 1000, 100, and 10. Cross sections were calculated with three orientations of the plume relative to the radar. The orientations were normal to the plume axis, along the plume axis, and at 45° to the plume axis. Four radar frequencies were considered: 200 MHz, 400 MHz, 3GHz, and 10GHz. The results

of the calculations are given in Fig. 10. Where finite volume corrections were made, we see this indicated on the figure by an asterisk. Finite volume corrections are needed when the value of ka exceeds 1. Normalized scattering cross sections vary from a high value of 2660 to a low value of 0.03, depending on the parameters considered. The cross section can be large for some parametric assumptions about the nature of the plume turbulence.

PLUME CALCULATION AND ELECTROMAGNETICS FOR A SURFACE SCATTERING PROBLEM

The same two computer programs were used to calculate the electron distribution in the plume for the surface scattering problem as for the volume scattering problem. The reaction and rate constants used, however, were for the solid propellant. The nozzle program also predicted the formation of some aluminum oxide in the exhaust products. This material will form a particulate matter, 95 percent of which will be less than 10 microns diameters in size. As these particles flow from the combustion chamber and out the nozzle, their inertia prevents them from spreading radially outward to the edge of the nozzle. The heavier particles remain close to the axis of the nozzle. The study also shows that the particle density at the exit plane is relatively sparse, varying from a high value of 2.2 particles/cc on the centerline for the

1.5 micron size to about 0.018 particles/cc at a distance of about half an exit plane radius for the 8-micron size. One would expect these low density and particle inertia effects to continue out the nozzle. Therefore a relatively narrow cone of low density particulate material would be seen extending out into the plume. Since the particulate matter is a dielectric and its number density is very low throughout the plume, the electromagnetic scattering cross section of this material will be very low. Therefore, electromagnetic scattering from the particulate matter is neglected in this investigation.

The alkali metal concentration in the solid propellant was about 200 ppm and this made the electron concentration in the plume about 20 times greater than that of the liquid propellant calculation. We are now interested in the scattering cross section for 400 MHz waves. These two facts combined to make a large axisymmetric overdense region in the plume. This volume expands rapidly to 4.6 meters in diameter close to the nozzle exit plane, then bows out gradually to 6.5 meters and finally tapers to 2.6 meters in diameter at 150 meters from the nozzle exit plane. The average plasma frequency in the overdense volume is 10^{10} radians/second and the radar frequency is 2.5×10^9 radians/second. Since the overdense volume is so large for the solid propellant rocket at this altitude, we assume that the reflection from this fluctuating

surface is the dominant process in this analysis. A sketch of the overdense volume as actually computed by the exhaust plume program is shown in Fig. 11.

Based on the calculated overdense volume described above, the following analytical model for the surface scattering is proposed. The overdense volume is approximated as a cylinder of 150 meters in length and 4.6 meters in diameter. Corrections to the scattering cross section are then made for a finite conductivity plasma and for surface roughness due to turbulence. These corrections are made by multiplying the smooth infinite conducting surface cross sections by correction factors for finite conductivity and surface roughness. Thus the corrected expression for the cross section would be

$$\sigma_c = (\sigma_{sc}) \cdot (C_{fc}) \cdot (C_R) \quad , \tag{16}$$

where σ_c is the corrected cross section, σ_{sc} the smooth cylinder cross section, C_{fc} the finite conductivity correction and C_R the roughness correction. The analytical expression used is for the backscatter cross section at broadside for a cylinder where the length and radius are large compared to the radar wavelength. The cross section expression for a smooth cylinder is given by the following expression (Ref. 9):

$$\sigma_{sc} = 4\ k_o R t^2 \quad , \tag{17}$$

where R is the cylinder radius and t is the half length of the cylinder.

The finite conductivity correction is based on a planar plasma-free space interface. Finite conductivity arises as a result of collisions of electrons with neutral particles in the plume. The average collision frequency in the overdense region of the plume is 1.2×10^{10}. One method for correction of the finite conductivity effects is proposed by Hochstim (Ref. 10). He calculates the reflection coefficient of a semi-infinite ionized gas medium with a plane boundary. Figure 12 is taken from Hochstim's report. It can be seen that, for the conditions of this problem ($\nu_c = 1.2 \times 10^{10}$, $\omega_p = 10^{10}$, $\omega = 2.5 \times 10^9$), the reflection coefficient is about 0.32. Since the cross section correction is proportional to the square of the reflection coefficient, the finite conductivity correction factor (C_{fc}) would be 0.1.

Hochstim's analysis is for a semi-infinite medium. French, Cloutier, and Bachynski (Ref. 11) have calculated the reflection coefficient for the case of a slab 2.5 plasma wavelengths thick and also for a semi-infinite slab. For the conditions of our plasma ($\nu_c/\omega_p = 1$) there is little difference between the reflection coefficients for the two thicknesses given above. Only for small values of ν_c/ω_p (less than 0.1) where there are wide

periodic variations in the reflection coefficient as ω/ω_p increases for the plasma of thickness $2.5\ \lambda_p$ would it be possible to have large differences in reflection coefficient for the thick and thin slab cases. For the average plasma and collision frequencies of the overdense volume, French et al. determine a reflection coefficient of 0.3 or a correction to the cross section of 0.09. Since the thin slab and semi-infinite volume finite conductivity corrections are almost the same value, one could assume that the correction factor for the cylinder is 0.1.

The second correction factor to be applied to the cross section for the smooth cylinder is that of surface roughness. This roughness arises because of the turbulent nature of the flow and manifests itself as a moving, or undulating, surface where the motion of neighboring positions is statistical in nature. One analytical approach that one could take is to carry over to our problems those techniques applied to the scattering from rough solid surfaces. Application of these ideas to wakes was first proposed and qualitatively discussed by Salpeter and Treiman (Ref. 12). The analysis of the electromagnetics of scattering from rough solid surfaces can be categorized as two limiting classes of problems. The classes are called either slightly or very rough. If the RMS height of the surface roughness is much less than the radar wavelength, the surface is called slightly

rough. If the RMS height is much more than the wavelength it is called very rough.

For slightly rough surfaces one usually considers the scattering cross section from both the coherent (specular) and incoherent (diffuse) part. The coherent part is that which comes from a smooth surface. The incoherent part comes from the roughness itself. Slightly rough surface theory is derived as a perturbation analysis around the smooth surface case. For most problems the coherent part of the cross section turns out to be much greater than the incoherent part. For this slightly rough surface coherent cross section we have chosen a model derived by Peake and also by Davies and reported in the radar handbook (Ref. 13). We are not going to derive the slightly rough equation. However, the expressions are valid under the condition that

$$k_o h < 1 \tag{18}$$

where h is the RMS roughness height and k_o is the wave number. The correction factor is

$$C_R = \exp -(\sqrt{2}\, k_o h \cos \theta_i)^4 \quad . \tag{19}$$

The expression for the incoherent part of the scattered radiation is

$$C_R = A k_o h^2 \ell^2 \tag{20}$$

where the coefficient A is 4 for an exponential correlation function and 8 for a gaussian correlation function, and ℓ is

the surface height correlation coefficient of the surface roughness measured along the surface. We will see later that the slightly rough electromagnetic case is used only when the correlation length or turbulence scale is very small. For this case the incoherent scattered radiation is much less than the coherent, and therefore it will be neglected.

For increased size of the surface roughness compared to the radar wavelength one finds that the scattered power becomes less and less coherent until at some large RMS roughness height the coherent fraction of the scattered power disappears and all the scattered power is then incoherent. The surface is then called very rough. The very rough surface correction factors are valid under the conditions that

$$k_o h > 5 \quad . \tag{21}$$

The assumptions of the model are the following: a) the surface radius of curvature must be larger than the wavelength; b) isotropic roughness; c) the correlation length of the surface roughness must be less than the scattering area, and d) multiple scattering and shadowing are neglected. For backscatter with a gaussian surface correlation function the roughness correction factor is

$$C_R = \frac{\sec^4\theta_i}{S^2} \exp\left[- \frac{1}{S^2} \tan^2\theta_i\right] \tag{22}$$

where $S^2 = 4h^2/\ell^2$ and θ_i is an average angle of incidence. And for the exponential correlation function

$$C_R = \frac{3}{S^2} \sec^4\theta_i \exp\left[-\frac{6}{S^2} \tan \theta_i\right] \quad . \tag{23}$$

The turbulent fluid scattering surface is the physical area about which the least is known and therefore is the most difficult to model. Two different physical models come to mind (Fig. 13). Since the electron density decreases radially, it is possible that the surface where the plasma frequency is equal to radar frequency could be interior to the turbulent plume. The region interior to the scattering surface will be overdense. This model of the scattering surface is called the interior turbulent surface model. It is assumed that the surface turbulent properties such as the correlation lengths and functions would be the same as the volume properties described above. Therefore the RMS surface height and the correlation length along the surface are equal to the volume radial and longitudinal correlation lengths, respectively. Recent studies (Refs. 5, 6, 7, and 14) have shown that the interior turbulence is mainly isotropic. Therefore the correlation length normal to and on the surface of the turbulence will be equal. For the interior turbulence, calculations were made for plume diameter to correlation length ratios (R/a) of 10 (large correlation length) and 1000 (small correlation length). This corresponds

to correlation length of 0.97 and 0.0097 meters, respectively. For a 400 MHz wave, then, from expressions (18) and (21) the surfaces are slightly rough for the large correlation length and very rough for the small correlation length. For the interior turbulence model one would use Eq. (20) for the slightly rough roughness correction and Eqs. (22) and (23) for the very rough roughness corrections.

The second model of surface turbulence will now be considered. In many cases the scattering surface is the surface of the turbulent jet itself. This jet surface, called the intermittent front, is undulating in a statistical manner. The intermittency, a statistical parameter, expresses the average fraction of time that a region in space near the front is interior to the turbulent plume. There are two other measures of the intermittent front that are important for electromagnetic wave scattering (Ref. 15). They are the standard deviation of the intermittent front radius fluctuation (σ_f), which is analogous to the radial correlation length, and the apparent wavelength of the turbulent front (Λ_f), which is analogous the axial correlation length.

Demetriades (Ref. 15) has measured both of these quantities in a plasma jet experiment (Fig. 14). His jet was produced by passing argon gas through a dc arc and expanding it through a supersonic nozzle with a 2.5 cm exit plane diameter into a large vacuum tank

held at 10 torr pressure. Measurement of the above two quantities of interest was made with Langmuir probes and hot wire anemometers in the turbulent part of the jet.

Demetriades reported measurements of σ_f and Λ_f with respect to a transverse scale length L, which varies linearly with the axial position of the turbulent region of the jet. Furthermore, the mean diameter of the turbulent region of the jet increases approximately linearly with increasing axial position. The value of σ_f/L is 1.5 and the values of Λ_f/L vary between a maximum value of 15 and a minimum of 5. From this one can calculate a maximum and minimum value of $\sigma/\Lambda_f = h/\ell$ of 0.3 and 0.1, respectively. Since L and hence both σ_f and Λ_f scale linearly with the local diameter of the turbulent portion of the jet, we assume that $\sigma_f, \Lambda_f, h,$ and ℓ will scale linearly to the full size jet. From Eq. (21) we find that all the intermittent front cases are very rough surface cases. Therefore the surface roughness correction factor C_f is [illegible]ted from Eqs. (22) and (23). The value of $h/\ell = 0.1$ is ca[illegible]e large axial scale length case and $h/\ell = 0.3$ is called the small axial scale length case.

SURFACE SCATTERING FROM A SOLID ROCKET PLUME

Eight cross sections have been calculated for the two turbulent surfaces considered, namely interior and intermittent front

turbulent surfaces. All results are backscatter, broadside cross sections. The interior surfaces cases have been calculated for the large correlation length with both gaussian and exponential correlation functions and for the small correlation length. All interior surface cross sections consider isotropic turbulence only. For the interior surface case, the small correlation length turbulence leads to a slightly rough electromagnetic scattering model and the large correlation length to a very rough electromagnetic scattering model.

The intermittent front cases have been calculated for large radial correlation lengths only. However, calculations with both large and small axial-to-radial correlation length ratios and both gaussian and exponential correlation functions have been made. All the cases treated with the intermittent front models lead to the very rough electromagnetic scattering models. The cross section and type of all calculations have been summarized in Fig. 15.

CONCLUDING REMARKS

It has been fully recognized for many years that both the fluid and electromagnetic aspects of the study of rocket plumes and aircraft jets are very difficult areas of engineering physics. This is true for both the analytical and experimental work in the

field. In this paper we have presented the calculated radar scattering cross sections of a variety of analytical models of both underdense and overdense plumes. Some models give large cross sections, others small cross sections. The models include physical phenomena not considered in detail by previous investigators. However, the models presented by no means exhaust the field. One can think of many phenomena that might bear consideration in an analytical model. Two things can be noted, however. The first is that the "correct" model or models will have to be simple in order to be tractable. Secondly, the models must also explain or predict the scattering characteristics of plumes more accurately and in more detail than older, simpler models if the results are to be useful to the detection and discrimination systems of the future.

REFERENCES

1. Mikatorian, R. and Pergament, H., Aerochem Axisymmetric Mixing with Nonequilibrium Chemistry Computer Program, Aerochem Research Lab Inc., Princeton, New Jersey, Report No. TP-200, June 1969.
2. Booker, H. and Gordon, W., "Radio Scattering in the Troposphere," Proc. of IRE, Vol. 38, pp. 401-412, April 1950.
3. Tatarski, V., Wave Propagation in a Turbulent Medium, McGraw-Hill, New York, 1961.

4. Kodis, R., "Propagation and Scattering in Plasmas," Proc. of IEEE, Vol. 53, No. 8, pp. 1016-1024, August 1965.

5. Slattery, R. and Clay, W., "Measurement of Turbulent Transition, Motion, Statistics, and Gross Radial Growth behind Hypervelocity Objects," Physics of Fluids, Vol. 5, pp. 849-855, July 1962.

6. Clay, W., Herrmann, J., and Slattery, R., "Statistical Properties of the Turbulent Wake behind Hypervelocity Spheres," Physics of Fluids, Vol. 8, pp. 1792-1801, October 1965.

7. Guthart, H., Weissman, D., and Morita, T., "Measurement of the Charged Particles of an Equilibrium Turbulent Plasma," Physics of Fluids, Vol. 9, pp. 1766-1772, September 1966.

8. Luzzi, T., "Electromagnetic Wave Scattering from a Finite Volume Turbulent Plasma," IEEE Transactions on Antennas and Propagation, Vol. AP-17, pp. 342-347, May 1969.

9. Mentzer, J., Scattering and Diffraction of Radio Waves. Pergamon Press, New York, 1955.

10. Hochstim, A., Charts for Microwave Propagation through a Plasma, General Dynamics/Astronautics, Report No. AE62-0858, December 1962.

11. French, J., Cloutier, G., and Bachynski, M., "The Absorptivity Spectrum of a Uniform Anisotropic Plasma Slab," Can. Jour. of Physics, Vol. 39, p. 1273, 1961.

12. Salpeter, E. and Treiman, S., "Backscatter of Electromagnetic Radiation from a Turbulent Plasma," Journal of Geophysical Research, Vol. 69, No. 5, March 1964.

13. Ruck, G., Barrick, D., Stuart, W., and Krichbaum, C., Radar Cross Section Handbook, Vol. 2, Plenum Press, 1970.

14. Becker, H., Hottel, H., and Williams, C., "The Nozzle-Fluid Concentration Field of the Round, Turbulent Free Jet," Jour. Fluid Mechanics, Vol. 30, Part 2, p. 287, 1967.

15. Demetriades, A. and Doughman, E., Mean, Intermittency, and Turbulence Measurements in a Self-Preserving Plasma Jet, AIAA Paper 68-685, June 24-26, 1968.

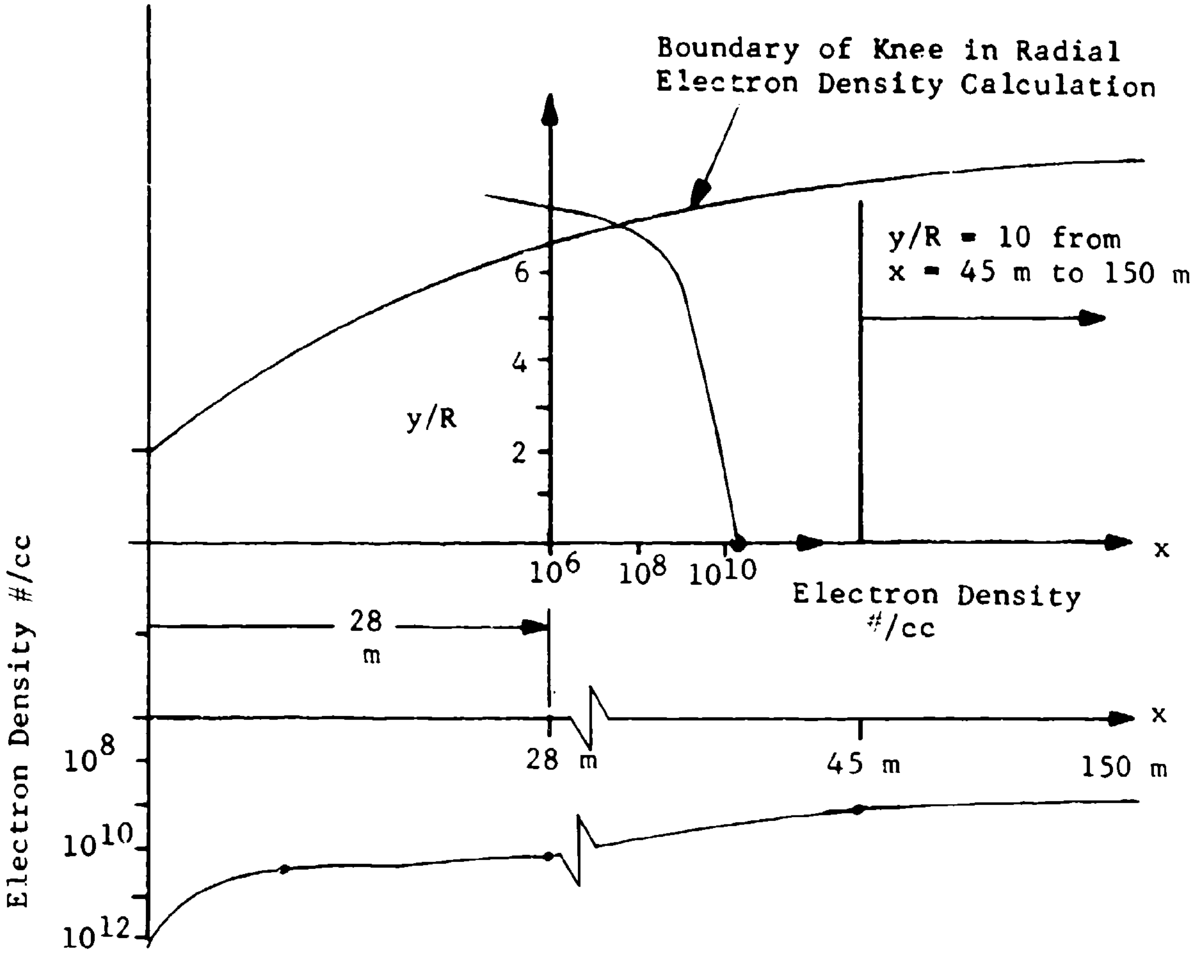


Fig. 1 Sketch of Plume Electron Concentration for Liquid Propellant Plume

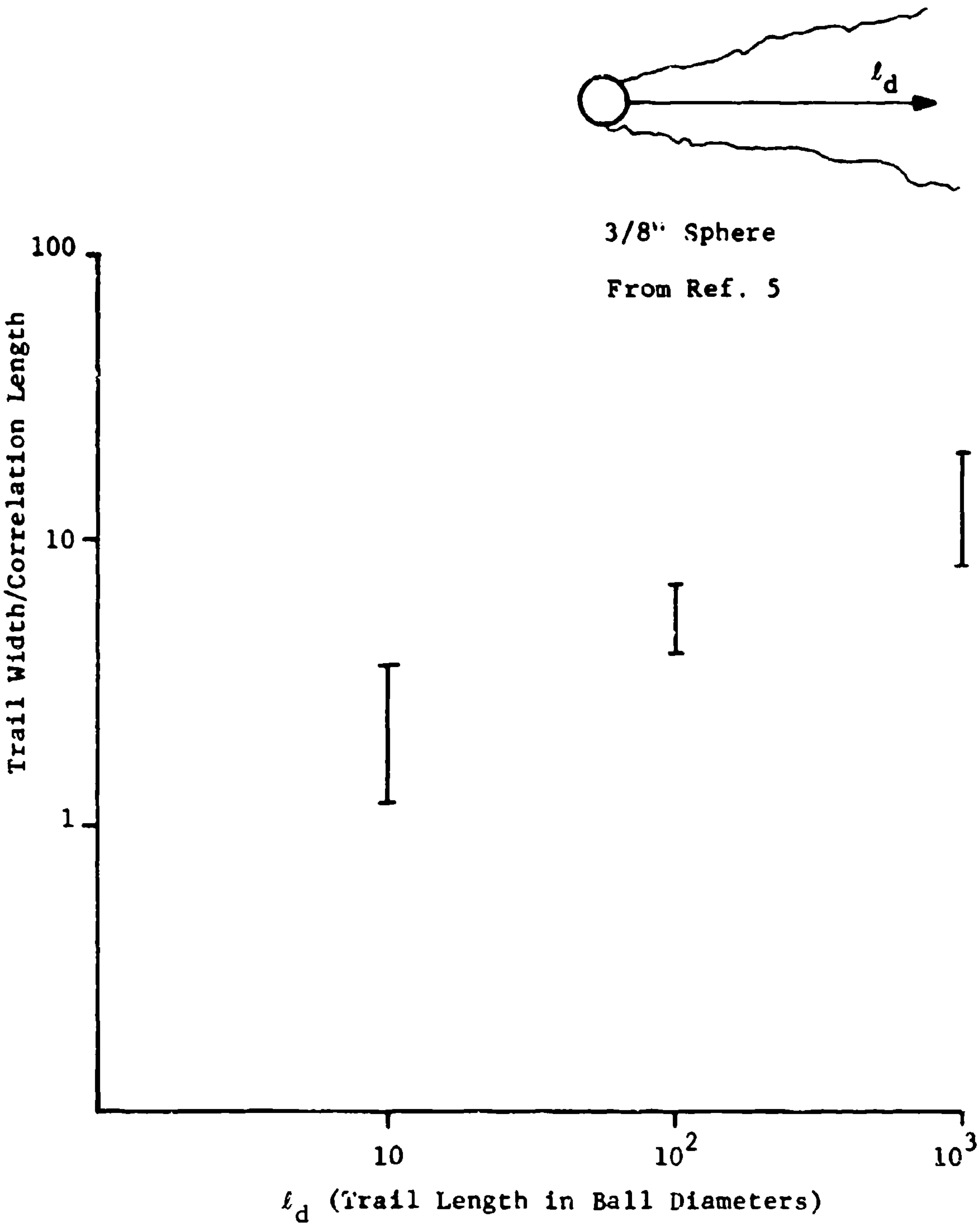


Fig. 2 Correlation Length in Wake of Sphere

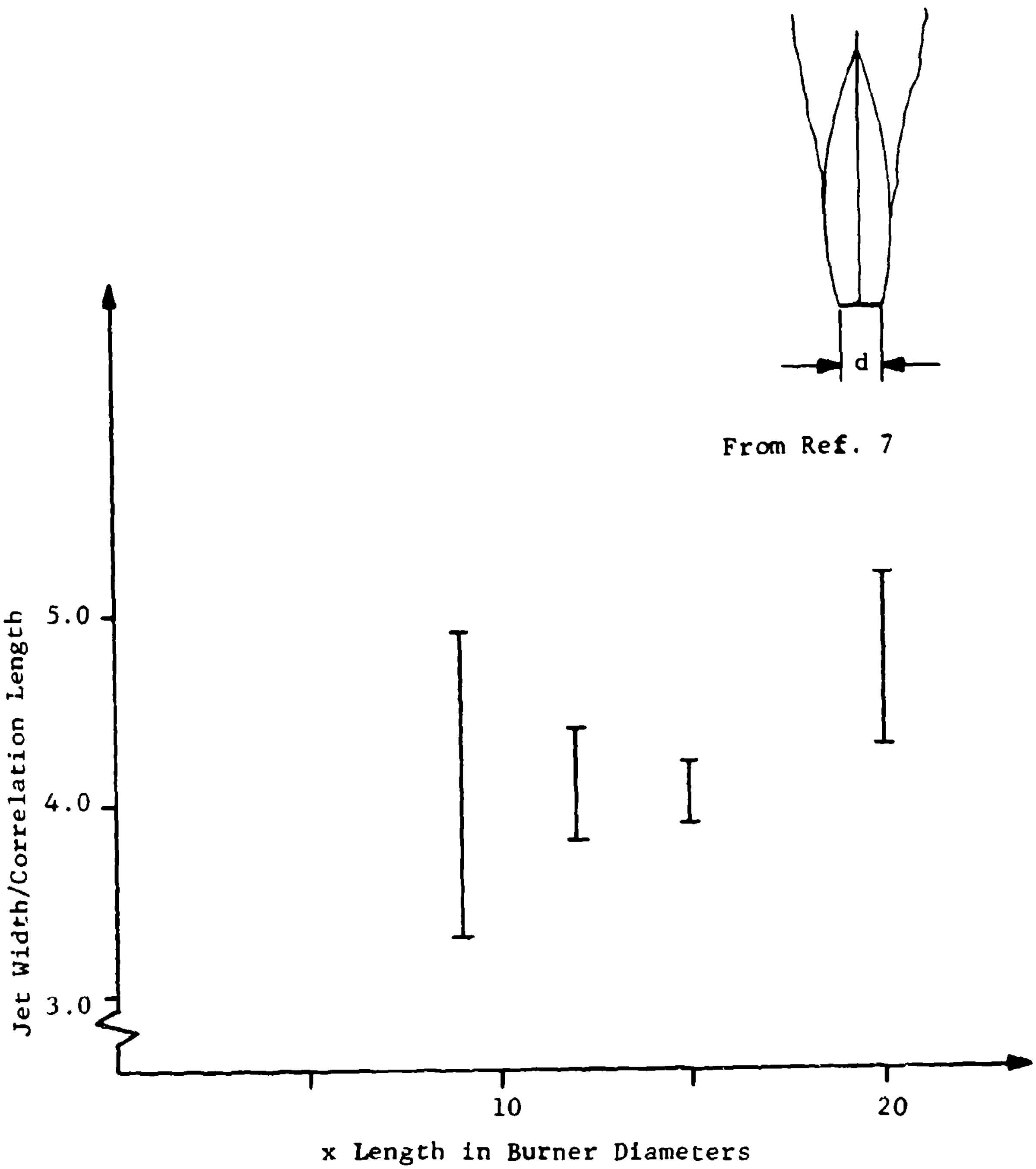


Fig. 3 Correlation Length in Flame

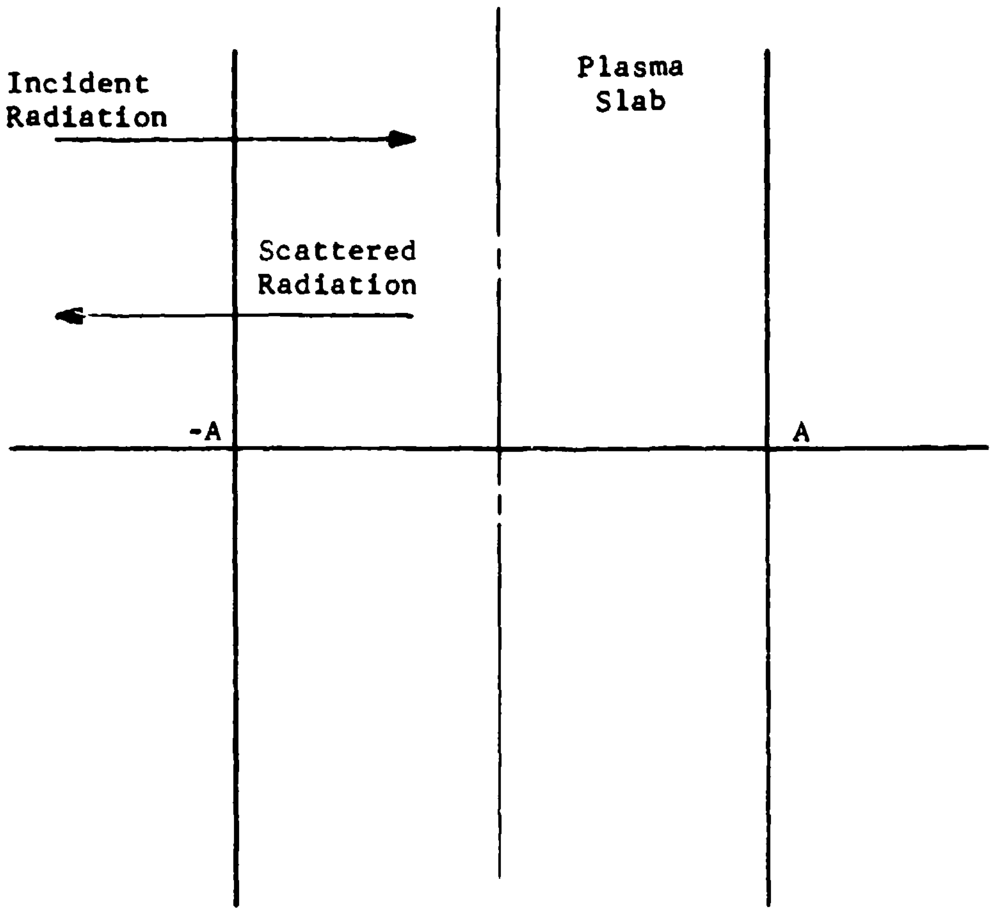


Fig. 4 One Dimensional Plasma (Volume Scattering)

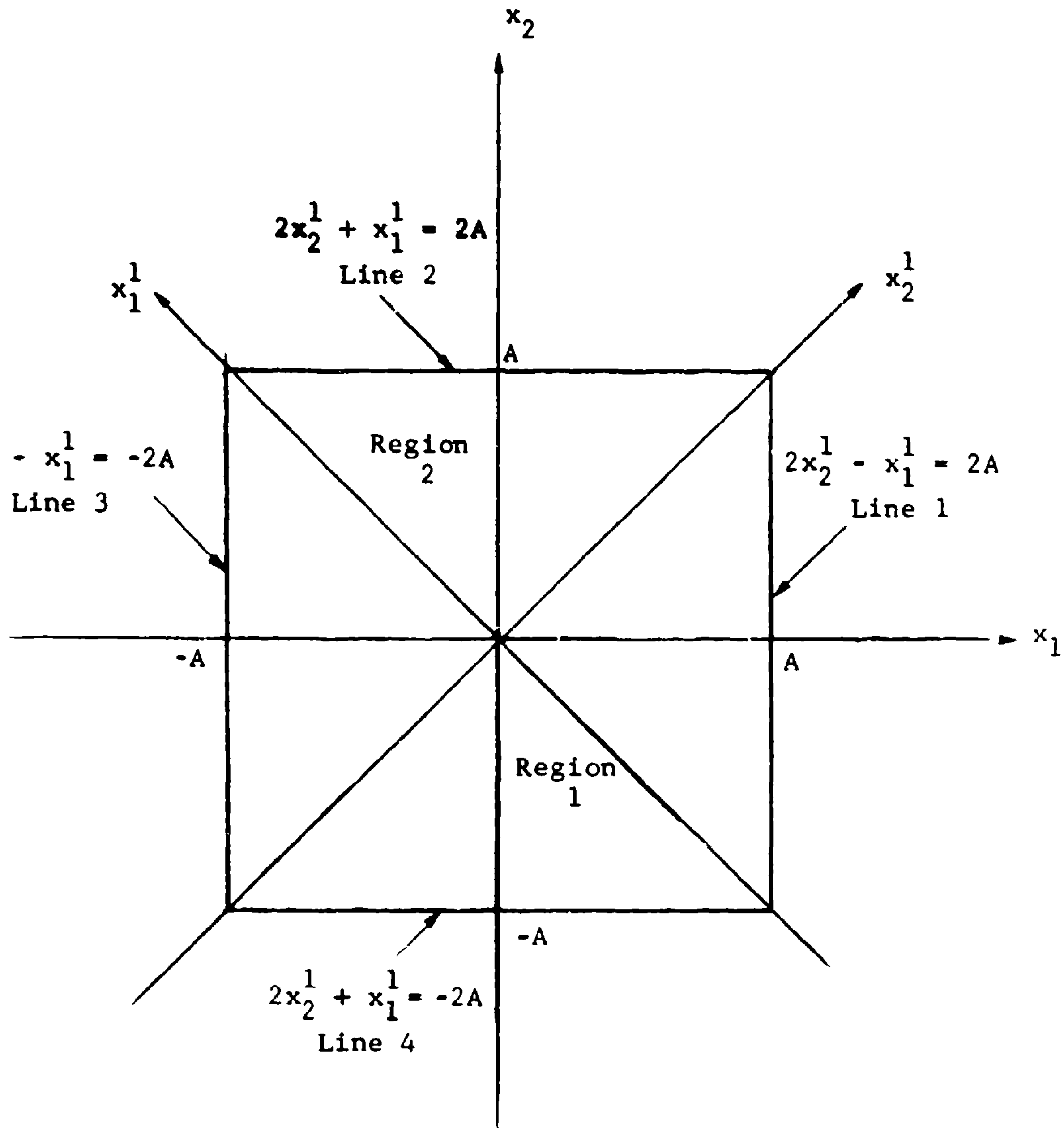


Fig. 5 Boundaries of Transformed x^1 and Real x Space for the One Dimensional Plasma

Variable	Lower Limit	Upper Limit	Index of I
x_2^1	$s_{112} - \frac{x_1^1}{2}$	$s_{121} + \frac{x_1^1}{2}$	$i = 1$
x_1^1	$-(s_{121} - s_{112})$	0	
x_2^1	$s_{111} + \frac{x_1^1}{2}$	$s_{122} - \frac{x_1^1}{2}$	$i = 2$
x_1^1	0	$s_{122} - s_{111}$	
y_2^1	$s_{212} - \frac{y_1^1}{2}$	$s_{221} + \frac{y_1^1}{2}$	$j = 1$
y_1^1	$-(s_{221} - s_{212})$	0	
y_2^1	$s_{211} + \frac{y_1^1}{2}$	$s_{222} - \frac{y_1^1}{2}$	$j = 2$
y_1^1	0	$s_{222} - s_{211}$	
z_2^1	$s_{312} - \frac{z_1^1}{2}$	$s_{321} + \frac{z_1^1}{2}$	$k = 1$
z_1^1	$-(s_{321} - s_{312})$	0	
z_2^1	$s_{311} + \frac{z_1^1}{2}$	$s_{322} - \frac{z_1^1}{2}$	$k = 2$
z_1^1	0	$s_{322} - s_{311}$	

Fig. 6 Limits of Integration for I_{kji} of the Scattering Cross Section

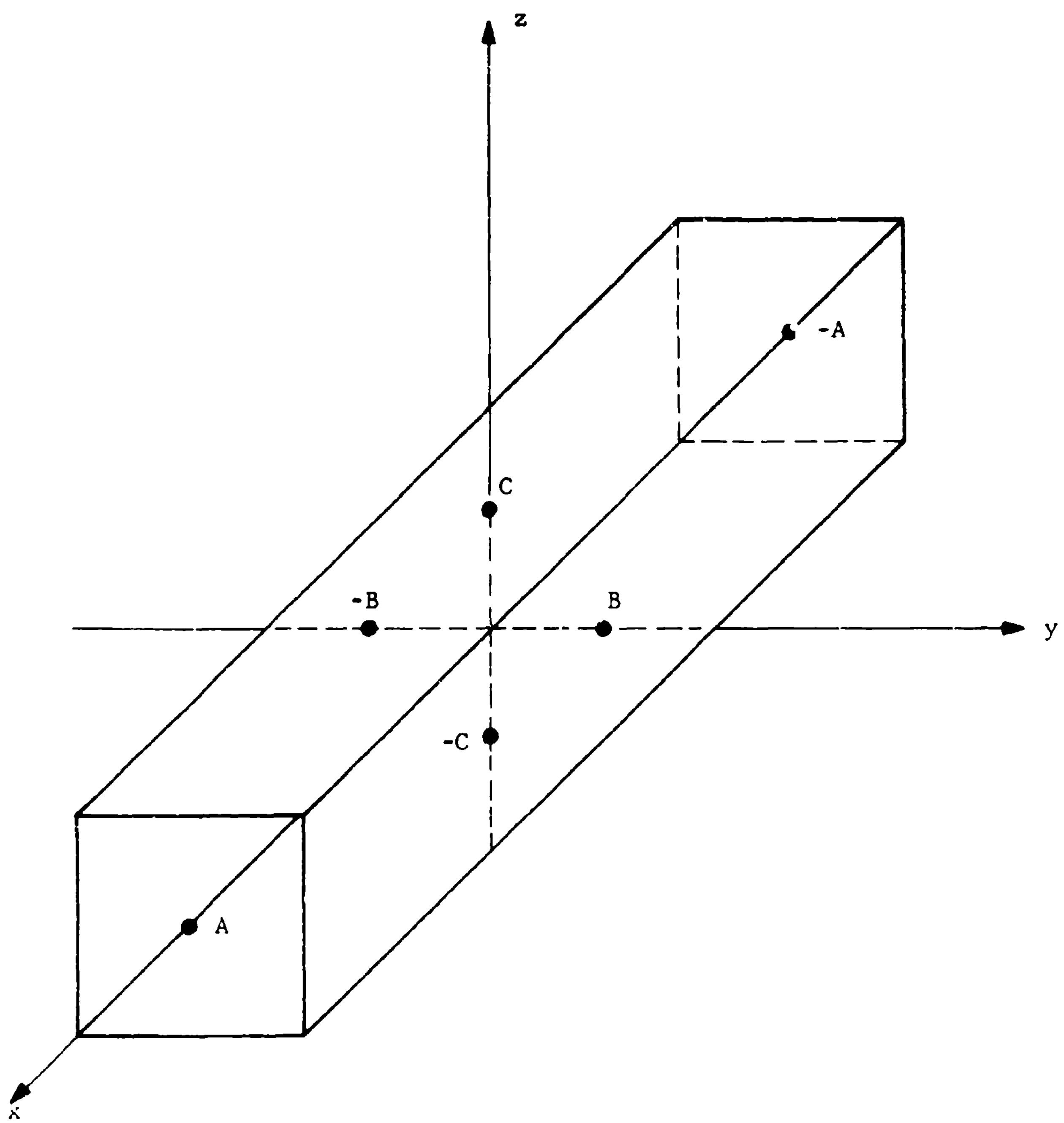


Fig. 7 Rectangular Parallelepiped Plasma

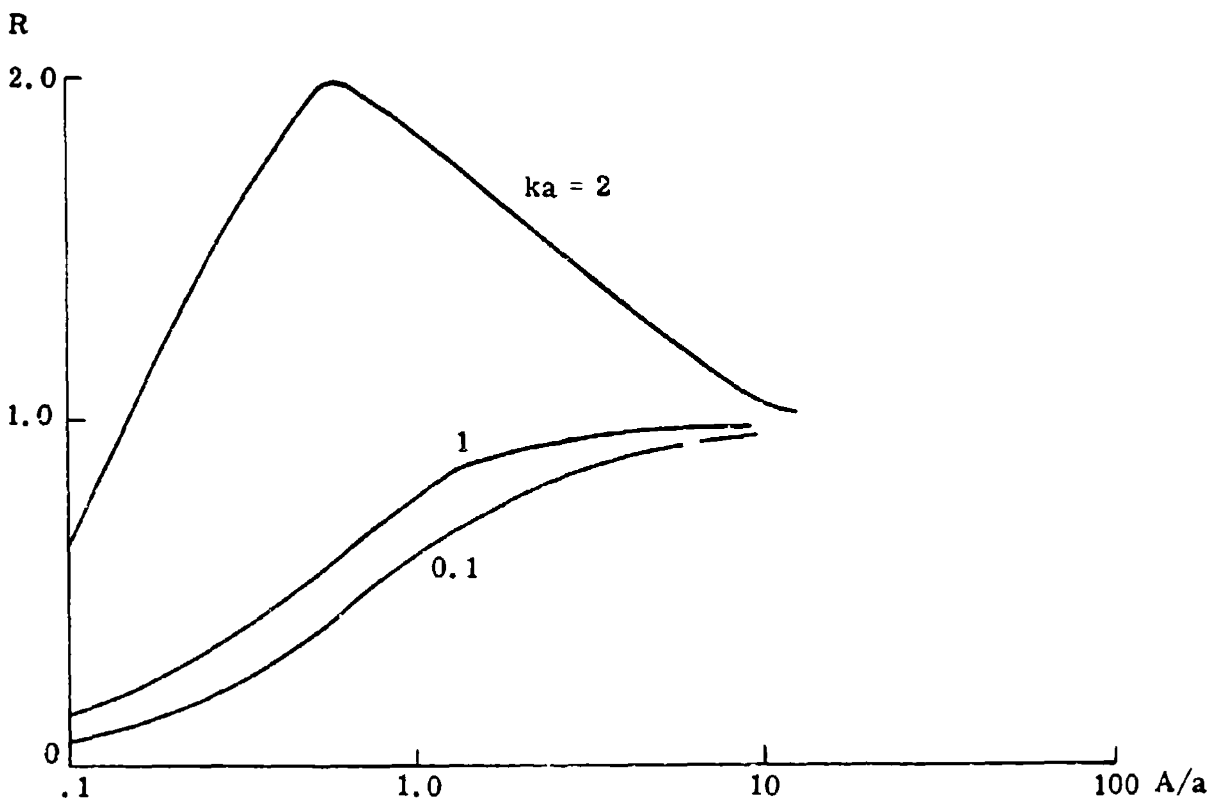


Fig. 8 Finite Volume to Booker-Gordon Scattering Ratio versus Plasma Dimension to Correlation Length Ratio for ka = 0.1, 1, 2

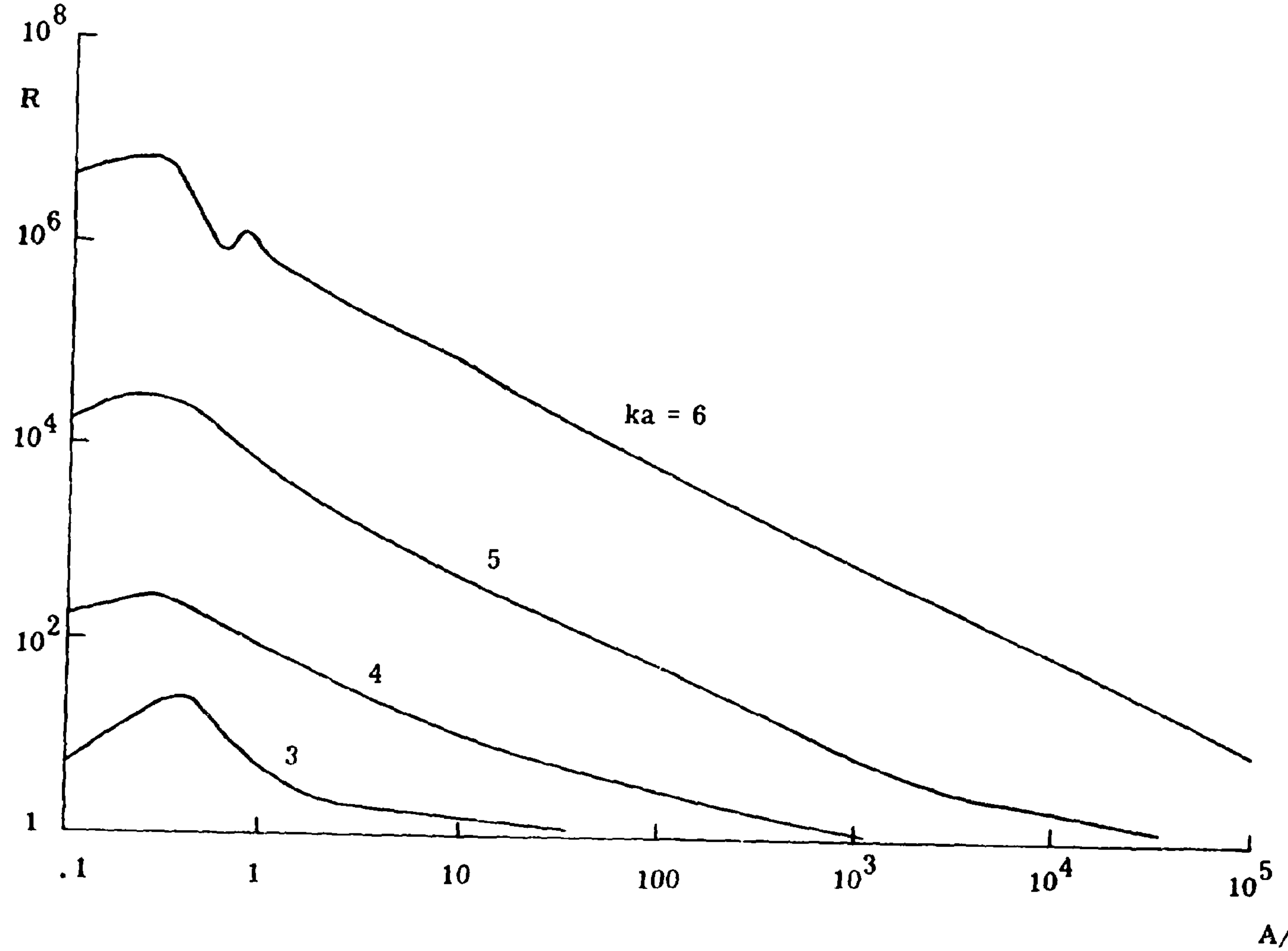


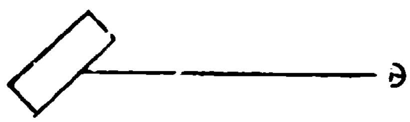
Fig. 9 Finite Volume to Booker-Gordon Scattering Ratio versus Plasma Dimension to Correlation Length Ratio for $ka = 3, 4, 5, 6$

Orientation: normal to axis

	R/a = 10	R/a = 100	R/a = 1000
200 MHz	2660.0 *	387.0	0.425
400 MHz		300.0	0.420
3 GHz		39.50 *	0.337
10 GHz			0.0328

Orientation: along axis

	R/a = 10	R/a = 100	R/a = 1000
200 MHz	186.0 *	387.0	0.425
400 MHz		300.0	0.420
3 GHz		3.95 *	0.337
10 GHz			0.0328



Orientation: 45° to axis

	R/a = 10	R/a = 100	R/a = 1000
200 MHz	77.5 *	387.0	0.425
400 MHz	0.0272 *	300.0	0.420
3 GHz		3.95 *	0.337
10 GHz			0.0328

*finite volume correction

Fig. 10 Normalized Cross Sections at 65000 ft Altitude

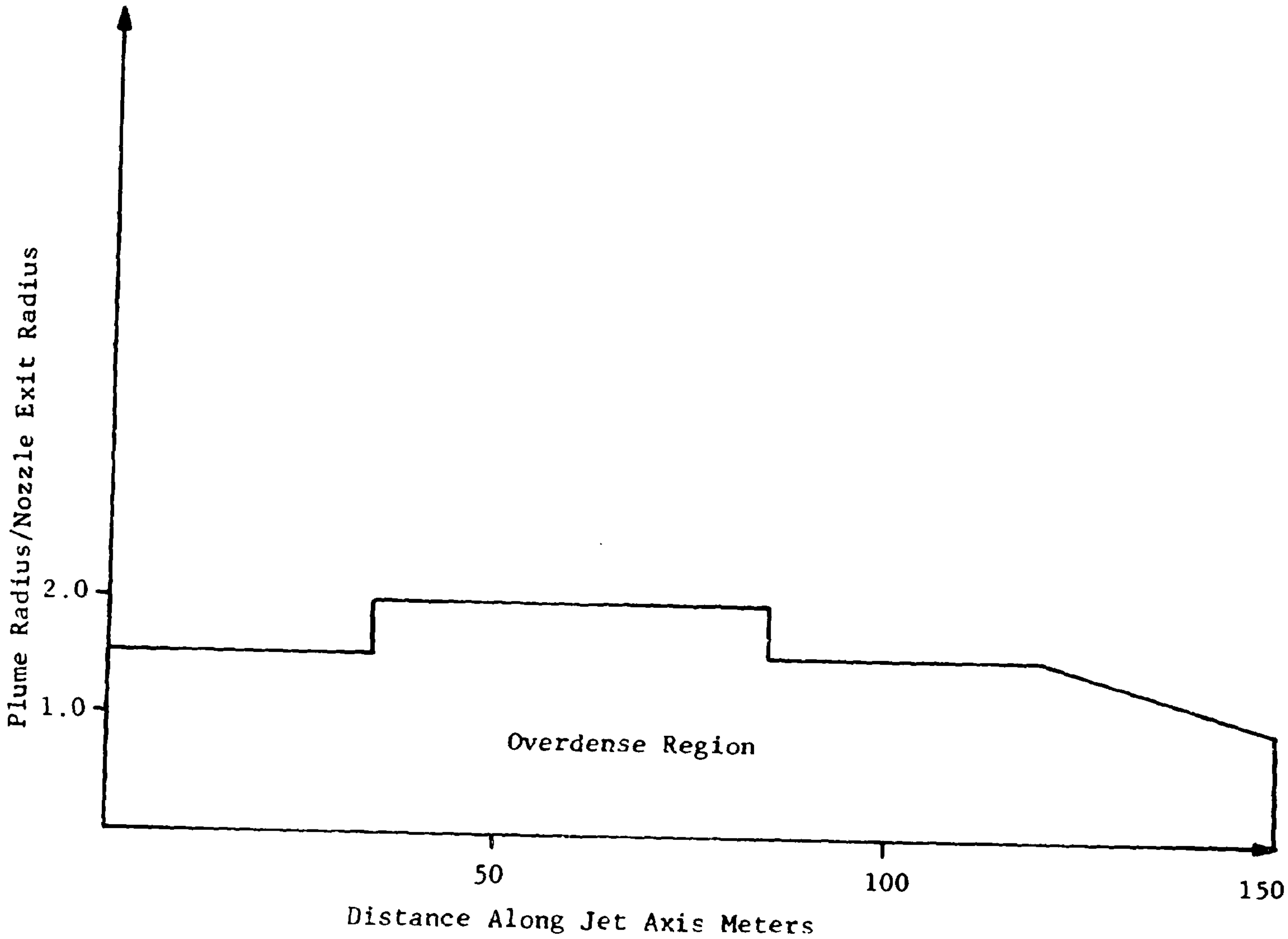


Fig. 11 Computer Profile of Overdense Region for Solid Propellant Plume

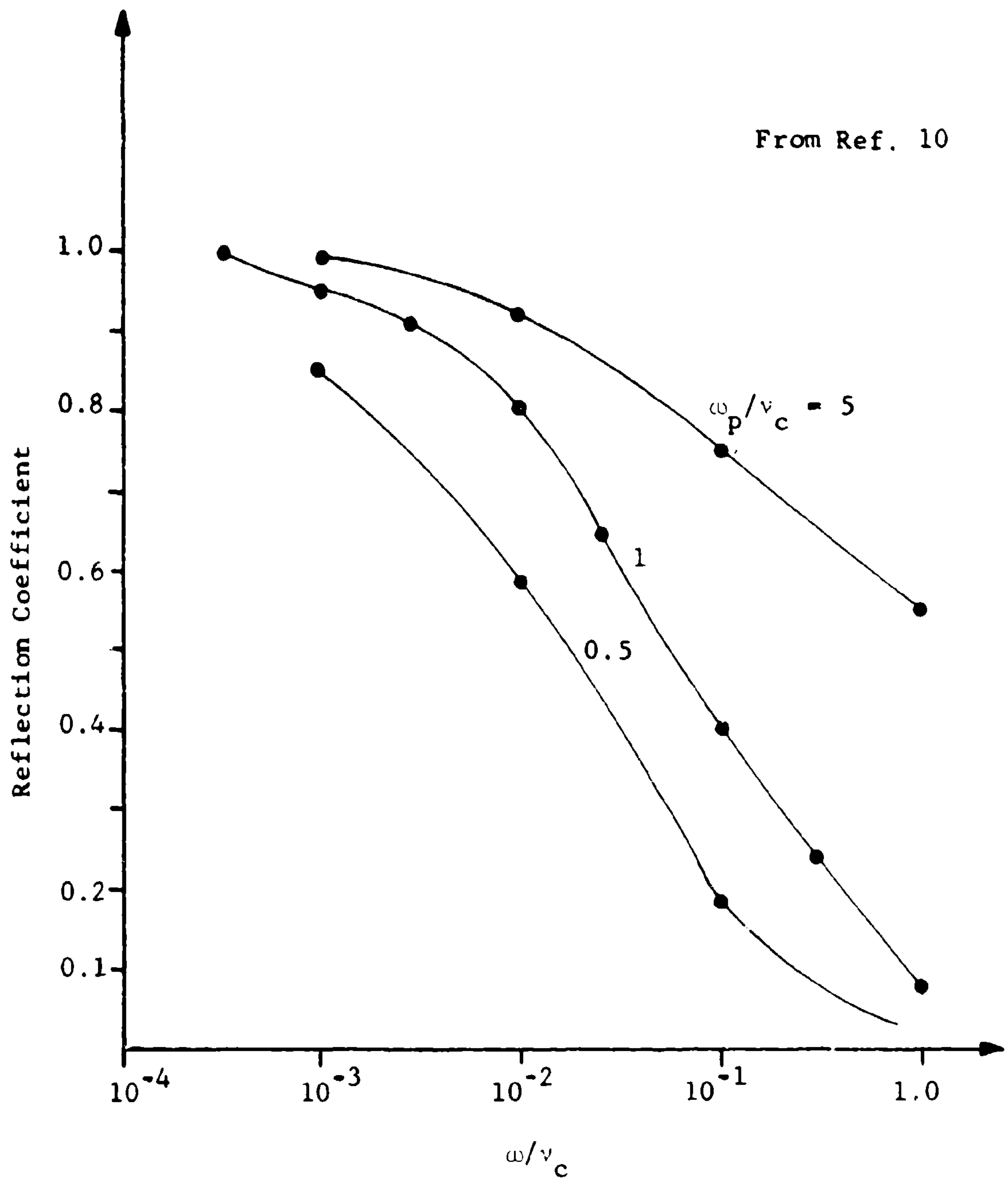


Fig. 12 Reflection Coefficient of a Homogeneous Semi-Infinite Plasma

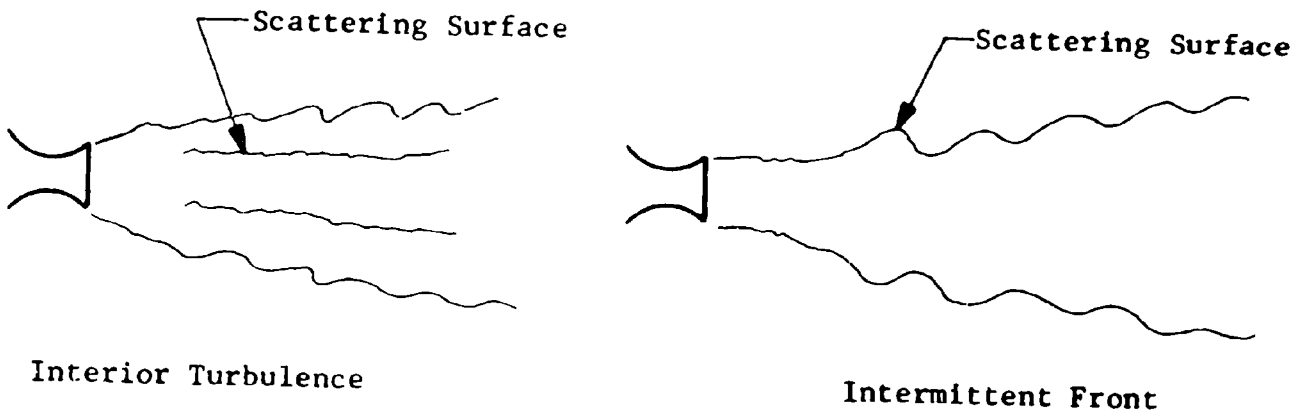


Fig. 13 Two Turbulent Surface Scattering Models

From Ref. 15

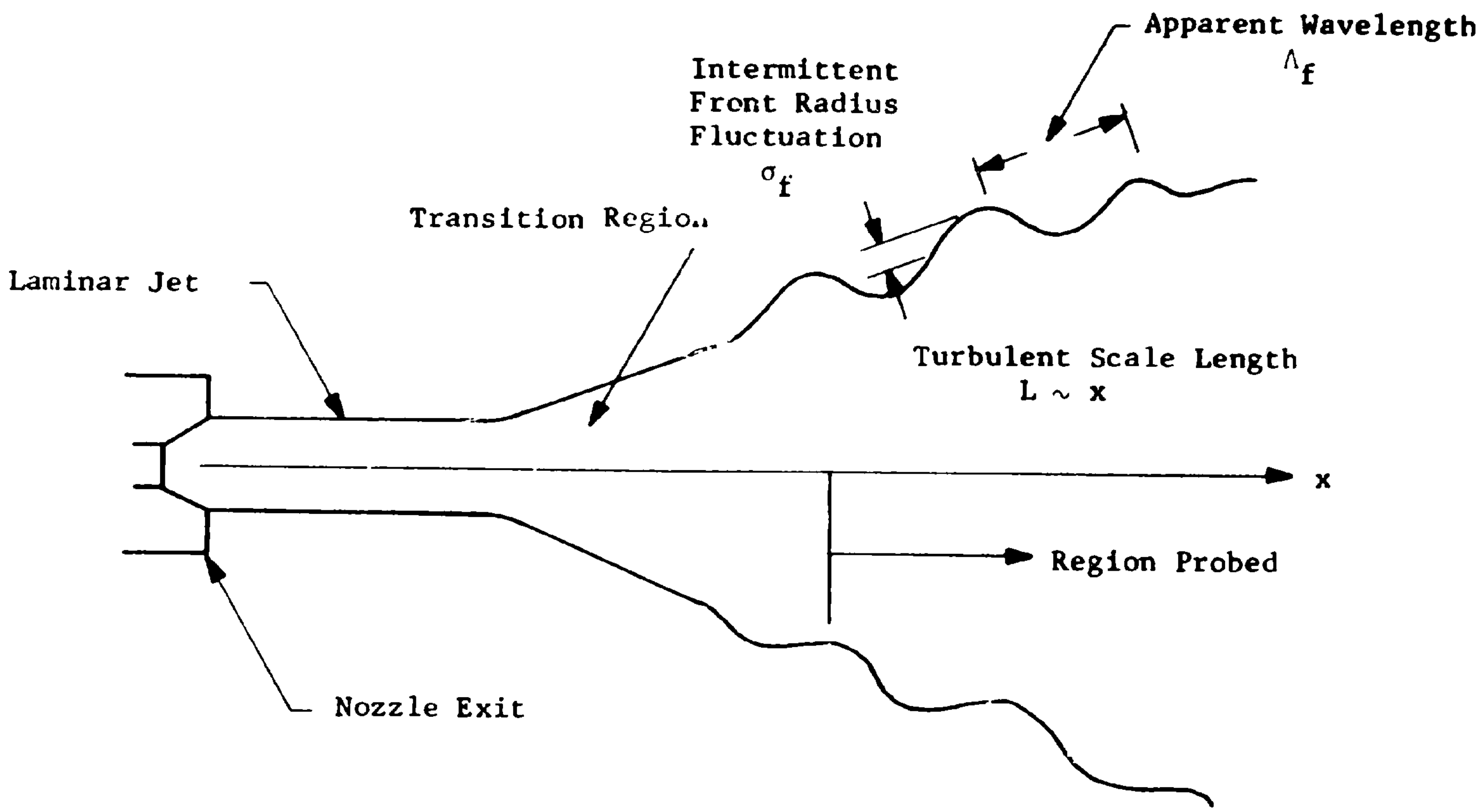


Fig. 14 Sketch of Plasma Jet Flow

		Interior Turbulence					
		Correlation Length		Correlation Function		Electromagnetics	
Roughness Correction	Normalized Cross Section	Small	Large	Gaussian	Exponential	Slightly Rough	Very Rough
	40,000	X		X		X	
		X			X	X	
.194	8050		X	X			X
.219	9070		X		X		X

		Intermittent Front (All Very Rough Electromagnetics)			
		Axial to Radial Correlation Length Ratio		Correlation Function	
Roughness Correction	Normalized Cross Section	Small	Large	Gaussian	Exponential
5.5×10^{-11}	2.1×10^{-6}		X	X	
6.06×10^{-5}	2.5		X		X
.0245	1010	X		X	
.020	826	X			X

Fig. 15 Roughness Corrections and Cross Sections for Various Surface Scattering Plume Model